

# Nonlinear Acoustics

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**Preface to the ASA Edition**

On this 10th anniversary of the publication of *Nonlinear Acoustics* by Academic Press, we are happy that the book has proven useful not only as a reference but also as a graduate level textbook at various universities. As a result of a merger, in recent years the book has been distributed by Elsevier. We are indebted to Elsevier for its release of the copyright in order for the Acoustical Society of America, as a nonprofit organization, to offer the present edition at a fraction of the previous cost and thus make it more accessible to students. We are also grateful to the Society for providing us with the opportunity to correct errors that were discovered in the first edition. Except for these minor corrections, the present edition is identical to the first.

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*May, 2008*

**Preface to the Original Edition**

This book is an introductory text on the theory and applications of nonlinear acoustics. For nearly 30 years the editors have, between them, taught a graduate course on this topic at the University of Texas at Austin. During this period the field has grown enormously. Many of the advances in theory have been inspired by measurements and applications. Because nonlinear acoustics now encompasses so many diverse areas, we decided that the best approach for a new book would be for individual chapters to be written by experts on their respective subjects. The book is meant to be a useful resource and reference for scientists and engineers and at the same time serve as a text for a graduate level course on nonlinear acoustics. Each chapter is written at a level and in a style that is oriented toward

classroom instruction. Moreover, consistent notation, insofar as practicable, and extensive cross-referencing between chapters have been used. Although we had graduate students in engineering and physics in mind, the material is accessible to anyone who is well grounded in the concepts of linear physical acoustics.

As chronicled in Chapter 1, the seeds of theoretical nonlinear acoustics were planted in the 18th and 19th centuries by the mathematicians and physicists who laid the foundations for fluid mechanics and wave motion. The principal contributions during this era are summarized in a benchmark article published by Lord Rayleigh at the beginning of the 20th century.<sup>1</sup> The current era of nonlinear acoustics may be traced to an innovative application referred to as the parametric array (chapter 8), the theory for which was presented in 1960 by P. J. Westervelt at the 59th meeting of the Acoustical Society of America (ASA).<sup>2</sup> Experimental confirmation of the parametric array was described by Bellin and Beyer in the very next paper.<sup>3</sup> Concurrently in the former Soviet Union, substantial progress in modeling the propagation of finite amplitude sound by using the Burgers equation was made by R. V. Khokhlov and coworkers.<sup>4</sup> These developments spawned a surge of related research in the 1960's, in response to which a symposium devoted exclusively to nonlinear acoustics was held in 1968 at the Navy Underwater Sound Laboratory, New London, Connecticut. The New London meeting has since been labeled the 1st International Symposium on Nonlinear Acoustics (INSA). Fourteen symposia in this series have been held, the last five in Japan (1984), the former USSR (1987), the US (1990), Norway (1993), and China (1996).<sup>5</sup> At the same time in the semi-annual ASA gatherings, papers on nonlinear acoustics have grown from just a handful at each meeting to enough for one or two full sessions per meeting.

The following guidance is suggested for those who wish to use the book as a course text. The first half of the book, Chapters 1-7, develops the physical concepts, mathematical models, and classical methods of solution that form the principal theoretical framework of nonlinear acoustics. Benchmark experiments are also described. These chapters, or at least portions of them, are appropriate as the core for an introductory course. The material is largely self-contained. In the interest of brevity, some equations are presented without derivation because they are derived in introductory texts on fluid mechanics (e.g., general conservation equations for thermoviscous fluids) and physical acoustics (e.g., various linear relations between acoustical quantities). For a text on fluid mechanics see the one by Landau and Lifshitz.<sup>6</sup> For acoustics see Pierce's book.<sup>7</sup>

The second half of the book, Chapters 8-15, covers special topics and applications, both theory and experiment. These chapters may be read in greater or lesser depth depending on the time available in a course and the interests of the students and instructor. Although independent of each other, they build upon the basic material presented in the first half of this book. Because of the complexity or breadth of various subjects that are covered, a number of sections in the second half are written as reviews. The chapter topics are indicative of current research areas in nonlinear acoustics.

The editors relied upon contributions and efforts from many individuals for the successful completion of this book. Allan Pierce is gratefully acknowledged for originally inviting the editors to prepare this book as a volume in the now discontinued series *Physical Acoustics* published by Academic Press. The authors are thanked for writing their chapters with student readers in mind, and for their patience with editorial changes requested for consistency and completeness. Yurii Il'inskii and Christopher Morfey provided invaluable advice on technical content. Former and current graduate students of the editors are in many ways responsible for the philosophy of this book through their responses to classroom lectures and questions associated with their research. Peggy Dickens was instrumental in preparing many of the figures. Finally, the tone of the book has certainly been influenced by the basic research in nonlinear acoustics performed by the editors and their students, most of which was supported by contracts and grants provided by the Air Force Office of Scientific Research (DTB), the National

Aeronautics and Space administration (DTB), the Office of Naval Research (DTB and MFH), the National Science Foundation (MFH), and the David and Lucille Packard Foundation (MFH)

<sup>1</sup> Rayleigh, Lord (1910). Aerial plane waves of finite amplitude. *Proc. Roy. Soc. A* 84,247-284

<sup>2</sup> Westervelt, P.J. (1960). Parametric end-fire array. *J. Acoust. Soc. Am.* 32 (A), 934-935.

<sup>3</sup> Bellin, J.L.S., and Beyer, R.T. (1960). Experimental investigation of the parametric end-fire array. *J. Acoust. Soc. Am.* 32 (A), 935.

<sup>4</sup> Soluyan, S.I. and Khokhlov, R.V. (1961). Propagation of acoustic waves of finite amplitude in a dissipative medium. *Vestn. Mosk. Univ. (Series III), Fiz. Astron.* 3, 52-61. See also Rudenko, O.V., and Soluyan, S.I. (1977); *Theoretical Foundations of Nonlinear Acoustics* (Plenum, New York). For earlier work on the Burgers equation, see Chapter 4, Section 5.1 of this book.

<sup>5</sup> For a listing and description up through the 12th ISNA, see Hamilton, M.F., and Blackstock, D.T., eds (1990). *Frontiers of Nonlinear Acoustics: 12th ISNA* (Elsevier, London).

<sup>6</sup> Landau, L. D., and Lifshitz, E. M. (1987). *Fluid Mechanics*, 2nd edition (Pergamon Press, New York).

<sup>7</sup> Pierce, A. D. (1989) *Acoustics—An Introduction to Its Physical Principles and Applications* (Acoustical Society of America, New York)

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Nonlinear acoustics is applied for damage detection. The study is focused on sensor location analysis with respect to vibro-acoustic wave modulations. The paper demonstrates that when structure is damaged, the high-frequency "weak" ultrasonic wave is modulated by the low-frequency "strong" vibration wave. As a result frequency sidebands can be observed around the main acoustic harmonic in the spectrum of the ultrasonic signal. However, intensity of modulation strongly depends on sensor location. Nonlinear implementations of acoustic diodes are inherently nonreciprocal and have received continuous attention from the beginning of the research boom for acoustic diodes. However, all the reported nonlinear schemes usually have the shortcomings such as low transmission ratio, action threshold, lack of stability and cumbersome setups. Figure 2 illustrates nonlinear saturation, caused by effective absorption of acoustic energy at the shocks for higher source amplitude. Saturation level is found to depend weakly on the linear gain  $G$  and it is about three times higher for the peak positive pressure, 2.2 times higher for the peak-to-peak pressure, 1.5 higher for the negative pressure, and 5 times higher for the intensity compared to the values, predicted by approximate analytic theory[1,5,6].

- Nonlinear acoustics in fluids
- Nonlinear acoustics in multiphase and porous media, and cavitation phenomena
- Nonlinear acoustics in solids and structures
- Nonlinear acoustics of reacting, relaxing media, and physical kinetics.
- Nonlinear acoustics in medicine and biology.
- Thermoacoustics.
- Nonlinear acoustics of atmosphere, ocean, and earth, and nonlinear underwater acoustics.
- Nonlinear aero- and hydroacoustics.
- Nonlinear acoustics and optics.
- General experimental methods.