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Making Sense of Word Problems

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Making Sense of Word Problems by L. Verschaffel, B. Greer and E. de Corte is the eighth book published in the series *Contexts of learning* (*Classrooms, Schools and Society*) by Swets & Zietlinger Publishers. The focus of the monograph is on students' unrealistic considerations when solving arithmetical word problems in school mathematics conditions. The contrast between the student's superficial "pure mathematical" approach to word problems on the one hand and the realistic consideration connected with their real-life experience and knowledge on the other hand is documented and analysed; proposals for ways of improving the situation are stated. All considerations are based on experiments carried out in several countries as well as theoretical background. The book is the result of several years of research conducted by the authors together with related research from various other countries.

The theoretical background to word problems in the book concerns the typical form of word problems, their components, functions and solving process. Criticism focuses on the fact that the current practice with word problems in school mathematics does not support students' tendency to use their everyday life knowledge. This "suspension of sense-making" is later documented and analysed in the whole monograph from several perspectives: ascertaining studies, intervention studies, the position of students, teachers and prospective teachers. It affects all mathematics education regardless of the culture, school level, level of involving new technology in mathematics education and social background. From this perspective, the monograph does not concern only the domain of solving word problems, but has a more global character. It concerns the culture of the mathematics classroom - the learning environment.

In the book, numerous examples of unrealistic answers to word problems from both literature and the authors' own observations are presented. Some of them are nonsensical ("How old is the captain?"), others have a very realistic background (e.g. "How much to post a letter?“ or "How many buses are needed"?). All these examples illustrate the fact that for many students, school mathematics has no connection to their real-life experience. When solving an arithmetical word problem, they simply apply the arithmetical operations algorithmically with neither realistic considerations nor the use of their common sense. The main reasons for such behaviour are the stereotyped nature of word problems typically presented in school mathematics and the rules for the "word problem game" introduced in the school practice.

The need to look closely at the observed gap between the learning of mathematics in school and its use compared with the realistic/non-realistic answers of students to the word problems containing some aspects of reality, led the authors to collect empirical data in a systematic way. In the first two studies, pupils were administered a paper-and-pencil test consisting of matched couples of items in the context of a typical mathematics lesson. Each pair of items consisted of an S-item (which can be solved applying an obvious arithmetic operation with the given numbers) and a P-item (which can be solved correctly only within the context and solver's common sense). In the experiments, both the students' realistic answers and the unrealistic ones accompanied by a realistic comment are labelled as "realistic reaction". These two studies were replicated by six studies in other countries, some of them being partly enlarged by the analysis of the influence of small variations in the presentation of P-items or in the tests (general hints, varying test presentation, various minimal interventions and different scaffolds). In two of them, the paper-and-pencil tests were accompanied by individual interviews. The results showed that the lack of relationships to reality is not necessarily the general solver's style, it was often connected only with their beliefs about school mathematics problem solving. The findings from these studies indicated that the majority of students, when solving arithmetic word problems in the school environment, demonstrated the tendency to apply one (or more) arithmetical operation(s) to the assigned data without a realistic consideration about the context. This tendency was not influenced by small variations of experimental settings.

Increased authenticity of the experimental settings had more influence on the student's solutions: that is changed task presentation towards a more authentic version together with students making realistic considerations; and/or modelling and solving mathematical application problems in the contexts familiar to students. The research confirmed that, due to the personalisation, solvers are more successful when the word problems relate to themes and contexts originated around familiar events, people or activities to the students.

Students' beliefs about school mathematics word problems, based mainly on their experience with textbook and school lesson word problems, support their superficial and artificial solving strategies. Standard forms of assessment mainly in the aspects of time, information, activity, social interactions, communication influence considerably affect the solvers' behaviour. Even teacher-students bring their beliefs acquired during their own schooling with them and behave similarly to children (though the ratio of realistic answers slightly increases compared to pupils). The rules of the "word problem game" cannot be studied without consideration of the most current teacher beliefs and behaviour with respect to word problems (selection of word problems dealt with, kind of class discourse provided, reactions to the unrealistic students' behaviour during the solving process, tests used etc.). There is no doubt that the situation needs a reform and this reform has already been started.

In the monograph, the authors do not only present investigative studies but they go further to intervention studies, from documenting the state of the student's confusion, to influencing it in the desired direction. There are several examples described leading to establishing a very different set of what is called by Cobb "socio-mathematical norms" or by Brousseau "didactical contract", namely realistic mathematical modelling and learning to solve mathematical application problems, anchoring mathematical problem solving in realistic contexts using information technology and improved assessment methods. These influences resulted in radically changed students' perceptions of word problem solving.

In the last part of the book, authors move from reporting the research findings to the discussion of philosophical and theoretical issues and analyses and the resulting suggestions for radical changes concerning the role of word problems within the curriculum. Of major concerns for mathematics educators is the discussion of how to construct word problems as exercises for mathematical modelling as opposed to routine applications of standard arithmetical operations. In the monograph, the relationship between the aspects of reality and mathematical structures is presented as one of the main factors influencing the unrealistic considerations. The authors characterise mathematics as having two sides, on the one hand description of the aspects of the world, on the other hand construction of abstract structures, the link between them being created by modelling. In traditional word problems (going many centuries back), mathematical procedures were applied to word problems without any links to realistic considerations. It needs a radical shift from uncritical application to modelling considering the reality of the situation described both in the curriculum and class discourse. In this modelling perspective, word problems can provide good or approximate models of situations or can be inappropriate in others.

One of questions behind all the considerations in the book is “What are word problems for?”. The authors divide word problems into two main groups – application of real-world problems on the one hand and artificial exercises providing frameworks for exploration of mathematical structures on the other, both of them having their irreplaceable position in students’ mathematics development. This division is not new, it is used in various studies, for instance Toom’s division into “applications” and “mental manipulatives”. From the linguistic point of view word problems are special types of texts. The linguistic frames form one of the central aspects of the word problem modelling process.

The last chapter of the monograph states the need and recommendations for reconceptualising the role of word problems in mathematics education. The key proposals are to adopt a modelling perspective, to increase the dialectical richness of interactions between teachers and students and among students and to modify the rules of the “word problem game”. The following key characteristics of the modelling approach to word problems are identified in the book: usefulness, not the absolute truth of models as the criterion for judging models, the evaluation of the usefulness of the model being relative to the goals of those developing and using them; the choice of model is influenced by the resources available; models always represent simplification of the reality, but it is necessary to keep this fact in mind; several competing alternative models can exist for a given phenomenon; modelling is an iterative process whose evaluation takes into account both the model and what is known about the modelled phenomenon. The authors summarise advantages and difficulties of the modelling perspective as well as new pedagogical purposes for which word problems can be used. For the proposed reform of framework within which mathematics is taught in schools, classroom culture has to be changed radically. Fundamental characteristics are: the use of more realistic, complex and challenging examples, explicit training in problem solving, collaborative work in small groups and whole class discussion, efforts to create a culture where students will develop appropriate beliefs about the nature of the modelling process, development of teachers’ conception of solving application problems and of appropriate way of teaching.

Evaluation

I find the book very valuable. It interconnects several allied researches dealing with the phenomenon of “suspension of sense making” conducted by authors from many countries throughout the world. The individual studies were studied as separate pieces of information. The book gives them a new dimension by their integration. The considerations are not valid only for word problem solving, they correspond with the findings of other studies from different school mathematics domains.

Many readers have experience of learned academic books which are difficult to understand because of the language used in them, of the background they rely on, of none coherent integration of various ideas etc. The reviewed monograph does not fall into this category. The language of the book is clear, precise and comprehensible for readers. It leads you into the text and ideas. Once I started, I wanted to continue to read to the end of the monograph. This effect is underlined by the nature of the studied phenomenon. It is not only a separate problem from a special historical, pedagogical, socio-cultural background point of view or of only the domain of solving word problems, but it has validity and consequences for human attitudes towards mathematics, towards overcoming obstacles during their whole life. Developing the ability to solve even non-standard problems and to link different domains together is enormously valuable.

The organisation of the text in the book is “reader friendly”. In the introductory part, the authors give the reader information about the theoretical background and basic terminology used throughout the book. Parts 2 and 3 begin with the summary of ideas presented in previous chapters accompanied with a brief description of the current goals. Each chapter begins with an interesting problem, quotation, or result illustrating the main problem dealt with. Presentations of studies contain all necessary information about the experiment design, conditions and the discussion and summary of their results. The findings are accompanied by schemata, pictures and other graphical representations, enabling the reader to grasp them more easily. The summaries and discussions of results at the end of each chapter are extremely helpful. The monograph contains fourteen pages of references, Name index and Subject index. The references represent a rich source of valuable papers and books dealing with word problem sense making. The rich reference part of the book enlarges the usability of the book.

It can be stated that the book is not only a report about very interesting and useful research but it is accompanied by proposals of how to improve the alarming situation of the gap between school mathematics and real life which can be seen all over the world. Also – and I find it extremely important as well – a perfect manual for beginning researchers giving the main rules of how to conduct their research.

The authors do not claim that they know the only successful remedy to restore the illness of "loss of common sense", they propose one of the possible ways for its improvement. An example of this fact is (and it is stated in several places in the book) that the authors do not propose rich modelling activities around complex, realistic problem situations (P-problems) as an alternative for doing straightforward word problems (S-problems). The different types of word problems (both S- and P-types) and different kinds of didactical contracts accompanying these different kinds of problems have their legitimate place in mathematics education. The difficult question for researchers and educators is to find a good balance between the two. It would be a mistake to interpret the educational implication of the book as a plea to simply replace standard word problems with ambiguous ones, wherein there must always be opportunities for the input of real-world knowledge that create difficulties for the relationship between a problem situation and a mathematical model.

The authors' proposal may (and I hope together with the authors that they really will) arouse a vehement discussion about the modelling perspective and the possibility of radical changes in school culture. It is one of the main features that make the book so interesting – there is no single and/or simple answer to the questions asked throughout the book and a wide discussion might help move the issue in a helpful direction.

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2. Problem: You analyzed the language before the lesson and picked some vocabulary which learners might find difficult. Butâ€¦they didnâ€™t. Instead, they started asking about absolutely different words. Last week, for example, we were talking about sport with my teens.Â Solution: if you are not sure about what words can make your students struggle, scan the task for potentially challenging lexis with the help of a dictionary or a special tool. Cambridge Dictionary usually refers to the level of words, and services like Oxford Text Checker use colour-coding to indicate words of different levels. It can make your assumptions more precise. Also, check the material for abstract concepts or tricky words which are difficult to be explained on spot. problem of suspension of sense-making amongst children in Singapore and the effects of an intervention programme. The responses from four intact classes were selected for initial analysis. The initial analysis aimed to help the.Â Figure 3 One intervention activity. The lack of sense-making when solving division-with-remainder word problems that were observed in many. previous studies (e.g., Cai & Silver, 1995) was not seen in the present study. Children in the present study were. When initially exploring bar modeling, word problems should be based on math concepts that are at least 2 years earlier than the grade level working with modeling so that students can focus on how the modeling works. The contexts should already "make sense" to the students so they can "make sense" of the modeling. Once the modeling makes sense to the students, it will give them a tool to "make sense" of the problem situation. Unifix Cubes can be a concrete representation to use as they begin this process; then the cube models can be drawn as tape diagrams to recor Word problems should be part of everyday math practice, especially for older kids. Whenever possible, use word problems every time you teach a new math skill. Even better: give students a daily word problem to solve so theyâ€™ll get comfortable with the process. Learn more: Teaching With Jennifer Findlay. 2. Teach problem-solving routines.Â One of the quickest ways to find mistakes is to look closely at your answer and ensure it makes sense. If students can explain how they came to their conclusion, theyâ€™re much more likely to get the answer right. Thatâ€™s why teachers have been asking students to â€œshow their workâ€ for decades now. Learn more: Madly Learning. 12. Write the answer in a sentence. When you think about it, this one makes so much sense. Pooling of Word Vectors Sense Inventory. Word Sense Induction. Figure 1: Schema of the word sense embeddings learning method. dense vector spaces with neural networks. First, contexts are represented with word embeddings and clustered.Â Our method can use existing word embeddings, sense inventories and word similarity graphs.Â We wanted to test if the same problem remains for a principally different method for similarity computation. Algorithm 1: Word sense induction. input : T â€“ word similarity graph, N â€". ego-network size, n â€“ ego-network connectivity, k â€“ minimum cluster size output: for each term t âˆˆ T , a clustering St of its N most similar terms foreach t âˆˆ T do V â† N most similar terms of t from T G â† graph with V as nodes and no edges E.